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A comparison between the PEP 1-1 model and the IFPRI standard model

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1. Foreword

The Poverty and Economic Policy Research Network (PEP) model and the IFPRI standard model have become very popular in the last years among the community of modelers. Both models have been widely used by economists throughout the world for policy analysis.

The PEP model is an extension of the widely used EXTER model and is the result of collaboration between Bernard Decaluwé, André Lemelin, Hélène Maisonnave and Véronique Robichaud from the PEP network. Given that there is a series of PEP models, we will focus here only on the static single country general equilibrium model called PEP1-1 (1 period – 1 country), which is comparable to the IFPRI model. It is a general model prepared for PEP members and other modelers for policy analysis at the national level.

The IFPRI standard model is a result of a joint work by Hans Lofgren, Rebecca Lee Harris, Sherman Robinson, Moataz El-Said and Marcelle Thomas undertaken in 2002 (Lofgren et al., 2002) at the International Food Policy Research Institute (IFPRI)[1]. It is a standard general equilibrium model built with the aim of analyzing trade and food policy issues in developing countries.

The purpose of this document is to give a brief comparison between the two models, namely their main similarities and differences, particularly from a theoretical point of view. It is not a full description of these models. We highly recommend the reader to read the manuals and user guides which come with the models (see references in Appendix) in order to have a complete overview. We neither discuss in detail the presentation strategy adopted in the manuals (such as the mathematical derivation of the equations) nor the different files that constitute the models. The reader is also encouraged to visit the websites[2] of the two institutions wherein he may find some training materials about the models and a series of studies which use them.

We first present the similarities between the two models and then highlight the main differences. We then provide a table which summarizes these results. In the appendix we present the full list of equations for both models.

[1] IFPRI is working on a new version of the model which will come soon. Given that this new version is still under construction, we will not focus on it here.

[2] http://www.pep-net.org/programs/mpia/pep-standard-cge-models/ and http://www.ifpri.org/publication/standard-computable-general-equilibrium-cge-model-gams-0

2. Similarities

The two models present many similarities:

2.1.General structure

Both models are single country static general equilibrium models. They adopt a Walrasian approach: perfect competition in all markets, only relative prices matter and all markets clear.

2.2.Institutions

Both models include the same institutions: households, enterprises, the government and the Rest of the world.

Households receive income from the factors of production, transfers from other institutions (enterprises, the government and the rest of the world). They spend their income on commodities, pay taxes, make transfers to other institutions and save.

Enterprises may receive payments from factors and transfers from other institutions. They allocate their income to transfers to other institutions, taxes and savings.

The government collects taxes, consumes commodities and makes transfers to other agents.

The Rest of the world receives payments for imports, from transfers from domestic agents, from factors and pays exports and makes transfers to domestic agents.

2.3.Production

On the production side, both models assume a nested production structure allowing the user to work with several categories of factors. At the top level, output is a function of intermediate input and value added; at the base level, value added is a Constant Elasticity of Substitution (CES) function of primary factors.

Both models allow the user to handle industries producing more than one product (rectangular social accounting matrices).

In both models the user has also the possibility to choose between industry-specific or fully mobile factors.

2.4.Demand

On the demand side, both models assume a Stone-Geary utility function for households (Linear Expenditure System function for the demand for goods).

Both models also take into account trade and transport margins in the final demand price system.

2.5.International Trade

Regarding international trade, both models assume a "small country" with exogenous world prices.

The two models also assume the Armington hypothesis of imperfect substitutability between imported and domestic goods. On the export side, it is also assumed that exported and domestic goods are imperfect substitutes in production.

3. Differences

There are also many differences between the two models. However these differences although large in terms of numbers, are not so substantial from a theoretical point of view. We present below the main distinctions.

3.1.Production

The first difference between the two models is related to the functional forms used in production. In the PEP model, there is no possibility of substitution between intermediate input and value added at the top level. The model assumes strict complementarity between the two elements (Leontieff Technology). The IFPRI model is a bit more flexible to this extent: the user has the choice between a CES or a Leontieff function. Furthermore, while the IFPRI model has a two stage nested production structure, the PEP one assumes a three stage level. In the IFPRI model, at the top level, total output is a (CES) or a Leontieff function of intermediate input and value added while at the base level value added is a CES function of primary factors. In the PEP model, at the top level, output is a Leontieff function of total intermediate input and value added while at the intermediate level value added is a CES function of composite labor and composite capital. At the base level, composite factors are CES functions of the primary factors. Composite labor is a CES function of the different categories of labor (unskilled labor, skilled labor…) and composite capital is a CES function of the different categories of capital (machinery and equipment, land…).

3.2.Home consumption

Another difference between the two models relies on how they take into account home consumption for agricultural products. As home consumption might be very high in developing countries, the user should be aware of this issue. While home consumption is absent from the PEP model, the IFPRI standard model takes it into account by valuing it at its opportunity cost (the activity specific producer price excluding transaction costs). One should note however that

even if the model takes into account home consumption, it implicitly assumes that the underlying farm household model is separable (recursive).

3.3.Commodity demand for investment purposes

Regarding investment demand, while in the PEP model investment demand is modeled as a Cobb Douglas function, the IFPRI standard model assumes a base year quantity multiplied by a fixed (exogenous) adjustment factor. With the IFPRI model specification, when investment demand is made endogenous by relaxing the adjustment factors, one will get a change in the same proportion in demand in all sectors. So commodity demand for investment purposes is not sensitive to relative price changes.

3.4.Government expenditures

In both models the total government consumption is fixed. In the PEP model, the government consumption is modeled with a Cobb Douglas function. In the IFPRI model, the government consumption demand is modeled like the investment demand: a base year quantity multiplied by a fixed (exogenous) adjustment factor. So, when made endogenous, the demand for all sectors will change uniformly.

3.5.Transfers

In both models government transfers to non-government agents are indexed to the Consumer Price Index. In the PEP model transfers are fixed at their initial value and then will automatically vary with the CPI unless the latter is used as the model numéraire. The indexation may be total, partial or absent. Furthermore, in the PEP model transfers from households to the government are related to social program contributions and are then treated like taxes with a marginal rate different from the average rate.

3.6.Taxes

In the PEP model all the taxes collected by the government are modeled separately. In that sense it is more detailed than the IFPRI model if the user wants to report detailed results on this topic. Income taxes are modeled in a flexible way. All income taxes are a linear function of total income allowing the marginal rate of taxation to be different from the average rate. The IFPRI model is also very flexible when it comes to modeling tax rates although there is no difference between the marginal and the average rate of taxation. For each institution, the direct tax rate is given by the exogenous base rate multiplied by an adjustment factor for scaling plus an additive

rate change[3]. This specification allows the user to test different closure rules regarding the government (see Table 1 below).

3.7.Savings

The two models also differ in the way they model marginal propensities to save for institutions. In the PEP model, households' savings are a linear (affine) function of their disposable income, indexed to the consumer price index. This specification is interesting as it avoids undesirable results in case some households exhibit negative savings in the SAM (such as an increase in savings when income falls). In IFPRI model, savings are a fixed proportion of disposal income and the marginal propensity to save is modeled in the same way like tax rates. For each institution, the marginal propensity to save is equal to the base rate adjusted for scaling for some selected institutions plus an additive rate change. As for the tax rates, this specification also allows the user to test different closure rules regarding institutions.

3.8.International Trade

It is important to note that although both models assume the small country hypothesis, the PEP model assumes finite price elasticity for the world demand for exports. With this specification the exported product by the country is not a perfect substitute of the competing products on world markets and local producers cannot increase their world market share unless they offer a Free on Board price smaller than the world price[4].

3.9.Closure rules

One can also notice a few differences regarding the closure rules and how they are handled in practice.

In the PEP model, the default closure rule consists of savings driven investment, fixed current account, fixed current government expenditures, fixed stock variations, specific sector capital, perfect mobility of labor and a fixed nominal exchange rate. There are of course a lot of alternative closures that are possible and that the user may apply to the model. Furthermore, the way the tax rates and the saving rates are specified increases the number of options. However it is worth noting that the user needs to modify the GAMS codes himself when he wants to modify the default closure (with the exception of the capital market).

[3] Similar to a point change

[4] A similar specification can be found in Dervis, de Melo and Robinson (1982).

Table 1: Alternative closure rules for the IFPRI standard model

Government	Rest of the World	Savings-Investment	Factor market
GOV-1: Flexible government savings; fixed direct tax rates	**ROW-1:** Fixed foreign savings; flexible real exchange rate	**SI-1:** Fixed capital formation; uniform marginal propensity to save point change for selected institutions	**FAC-1:** Perfect mobility with exogenous factor supply
GOV-2: Fixed government savings; uniform direct tax rate point change for selected institutions	**ROW-2:** Flexible foreign savings; Fixed real exchange rate	**SI-2:** Fixed capital formation; scaled marginal propensity to save for selected institutions	**FAC-2:** Fully segmented markets
GOV-3: Fixed government savings; scaled direct tax rates for selected institutions		**SI-3:** Flexible capital formation; fixed MPS for all non-government institutions	**FAC-3:** Unemployment with fixed wage rate and endogenous factor supply
		SI-4: Fixed investment and government consumption absorption shares (flexible quantities); uniform marginal propensity to save point change for selected institutions	
		SI-5: Fixed investment and government consumption absorption shares (flexible quantities); scaled marginal propensity to save for selected institutions	

Source: Lofgren et *al*. (2002) and author.

In the IFPRI standard model, the default closure rule consists of investment driven savings, fixed foreign savings with flexible exchange rate, fixed volumes of government consumption, flexible government savings and perfect mobility of factors. Theoretically, the user has the choice between 90 closures that are all already programmed: 3 for the Government, 2 for the rest of the world 5 for the savings-investment balance and 3 for the factor market closure (see Table 1). This gives the user a great flexibility. For instance if one assumes fixed investment, one can assume either scaled or uniform marginal propensity to save point change for selected institutions.

One great difference between the two models is related to the absorption. The IFPRI model allows the user to handle more closures regarding the savings-investment balance. The model includes three equations defining total absorption, the ratio of investment to absorption and the ratio of government consumption to absorption. This specification gives flexibility to the user. It

makes it possible to share the adjustment burden to absorption shocks uniformly across all the components of absorption (SI-4 and SI-5 in Table 1) instead of being fully supported by households' consumption (SI-1 and SI-2 in Table 1) or by investment (SI-3 in Table 1).

Regarding the choice of the numéraire, one should note that the default numéraire is the nominal exchange rate for the PEP model while for the IFPRI standard model the user may choose between the consumer price index and the domestic price index.

3.10.Data

It is worth noting that the way the two models present Social accounting matrices is slightly different. For instance, in the PEP tradition, commodities appear twice in the SAM (for local supply and for export markets) in order to have exports at both producer and consumer prices. Although this is not an issue when calibrating the models, people from the PEP network and from francophone Africa might be more familiar with the PEP paradigm while other people would prefer the IFPRI model.

4. Conclusion:

To sum up, one can say that both models are built in the spirit of Walras within the same theoretical paradigm. The main differences between the two models are in the details, giving the user a great flexibility in the modeling strategy.

From a practical point of view, while in the PEP model, the user needs to write his own codes when he modifies the default options, in the IFPRI model all the options are already programmed. To that extent, modelers who are beginners and do not want to do extra programming might prefer the IFPRI model. Furthermore, the new version of the IFPRI model will have an excel interface allowing the user to define and run his own simulations without additional coding in GAMS.

It is also worth noting that both models are well documented. The PEP model comes with a detailed manual explaining the microeconomic foundations of the model with all the mathematical derivations on a step by step basis while the manual accompanying the IFPRI model tends to assume that the user is already familiar with those concepts. Therefore, we think that the PEP model and its accompanying documentation is more suitable for beginners. They offer the users both the mathematical and the economic foundations of CGE models in a rigorous way. The IFPRI model is more general and allows users to choose among many options. It

should be used by experienced users who are a bit familiar with CGE models, in order to avoid misinterpretations while combining different options. The role of these options (mainly closure rules) is discussed and illustrated in Laborde and Traoré (2012).

References

Decaluwé, B., A. Lemelin, H. Maisonnave and V. Robichaud (2009), *The Pep Standard Computable General Equilibrium Model, Single-Country, Static Version Pep-1-1*, second revised edition, Poverty and Economic Policy (PEP) Research Network, Laval, Canada.

Dervis, K., J. de Melo and S. Robinson (1982), *General Equilibrium Models for Development Policy*, Cambridge: Cambridge University Press.

Laborde, D and F. Traore (2012), "The sensitivity of CGE models to macroclosures: Evidence from the IFPRI Standard Model", *AGRODEP Technical Note*, forthcoming.

Lofgren, H., R. L. Harris and S. Robinson (2002): *A Standard Computable General Equilibrium Model in GAMS*. IFPRI Microcomputers in Policy Research 5, Washington DC.

Appendix A: Summary table of the main characteristics of the models

	Production	Prices	International trade	Demand
PEP 1-1	Aggregate output for each industry is a CET function of the goods produced by this industry Nested production structure Top level: output is a Leontieff function of total intermediate input and value added Intermediate level: Value added is a CES function of the composite labor and composite capital Base level: Composite factors are CES functions of the primary factors Intermediate inputs are strict complements	Final demand prices include trade margins Domestic prices for products that are traded are function of world prices, the exchange rate, and the taxes or subsidies	Small country assumption: exogenous world prices Finite price-elasticity of the world demand for exports Composite good is a CES function of domestic and imported commodity Total output is a CET function of domestic and exported products	Household maximize a Stone-Geary utility function (LES demand function) Investment expenditures of each commodity is a fixed share of total investment (Cobb-Douglas Function) Fixed government expenditures on goods and services Government expenditures of each commodity is a fixed share of total expenditures (Cobb-Douglas Function)
IFPRI	Aggregate marketed output for each commodity is a CES function of industry specific production of that commodity Aggregate production is the sum of marketed quantity and home consumption Nested production structure Top level: output is a (CES) or a Leontieff function of intermediate input and value added Base level: Value added is a CES function of primary factors Intermediate inputs are strict complements	Final demand prices include transaction costs/margins Domestic prices for products that are traded are function of world prices, the exchange rate, and the taxes or subsidies	Small country assumption: exogenous world prices Infinite price-elasticity of the world demand for exports Composite good is a CES function of domestic and imported commodity Total output is a CET function of domestic and exported products	Household maximize a Stone-Geary utility function (LES demand function) Fixed investment demand is a function of the base year quantity multiplied by a fixed adjustment factor Fixed government consumption is a function of base year quantity multiplied by a fixed adjustment factor

	Institutions	Equilibrium	Closure rules	Numéraire
PEP 1-1	Total household income is the sum of factor incomes, transfers from firms, from the government (indexed to the CPI) and from the rest of the world Household savings are a linear (affine) function of disposable income Firms income come from factor incomes and other institutions Total government revenue is the sum of revenues from taxes, factors and transfers from other institutions Fixed indirect and factors tax rates Income tax rates are linear (affine) functions of total income	All markets clear Total savings equal total investment (Gross fixed capital formation and stock changes)	Default closure: Savings driven investment Fixed current account Fixed stock variations Flexible real exchange rate Full employment of factors Perfect mobility of labor Sector specific capital Alternative closures regarding the Government, the savings-investment balance, the rest of the world and the factor markets up to the user	Nominal exchange rate
IFPRI	Total household income is the sum of factor incomes, transfers from firms, the government (indexed to the CPI) and from the rest of the world Marginal propensity to save is equal to the base rate adjusted by a factor for some selected institutions plus a point change Firms total income come from capital incomes and transfers Total government revenue is the sum of revenues from taxes, factors and transfers from other institutions Direct tax rate is given by the exogenous base rate multiplied by an adjustment factor for scaling for some selected institutions plus a point change	All markets clear Total savings equal total investment (Gross fixed capital formation and stock changes)	Default closure: Investment driven savings Fixed current account Flexible real exchange rate Fixed stock variations Full employment of factors Perfect mobility of factors 90 alternative closures (3 for the Government, 2 for the rest of the world, 5 for the savings-investment balance, 3 for the factor market; see Table 1) .	Consumer price index (the domestic price index may also be used)

Appendix B: The PEP 1-1 Model[5]

SETS

Industries and commodities

All industries: $j, jj \in J = \{J_1, \dots, J_j, \dots\}$

All commodities: $i, ij \in I = \{I_1, \dots, I_i, \dots\}$

Imported commodities: $m \in M \subset I; M = \{M_1, \dots, M_m, \dots\}$

Non imported commodities: $nm \in NM \subset I; NM = \{NM_1, \dots, NM_{nm}, \dots\}; NM \cap M = \emptyset$

Exported commodities: $x \in X \subset I; X = \{X_1, \dots, X_x, \dots\}$

Non exported commodities: $nx \in NX \subset I; NX = \{NX_1, \dots, NX_{nx}, \dots\}; NX \cap X = \emptyset$

Production factors

Labor categories: $l \in L = \{L_1, \dots, L_l, \dots\}$

Capital categories: $k \in K = \{K_1, \dots, K_k, \dots\}$

Agents

All agents: $ag, agj \in AG = H \cup F \cup \{GVT, ROW\} = \{H_1, \dots, H_h, \dots F_1, \dots, F_f, \dots, GVT, ROW\}$

Household categories: $h, hj \in H \subset AG = \{H_1, \dots, H_h, \dots\}$

Firm categories: $f, fj \in F \subset AG = \{F_1, \dots, F_f, \dots\}$

Non governmental agent: $agng \in AGNG \subset AG = H \cup F \cup \{ROW\} = \{H_1, \dots, H_h, \dots F_1, \dots, F_f, \dots, ROW\}$

Domestic agents: $agd \in AGD \subset AG = H \cup F \cup \{GVT\} = \{H_1, \dots, H_h, \dots F_1, \dots, F_f, \dots, GVT\}$

VARIABLES

Volume variables

$C_{i,h}$:	Consumption of commodity *i* by type *h* households
$C_{i,h}^{MIN}$:	Minimum consumption of commodity *i* by type *h* households
CG_i :	Public consumption of commodity *i*
CI_j :	Total intermediate consumption of industry *j*
DD_i :	Domestic demand for commodity *i* produced locally
$DI_{i,j}$:	Intermediate consumption of commodity *i* by industry *j*
DIT_i :	Total intermediate demand for commodity *i*
$DS_{j,i}$:	Supply of commodity *i* by sector *j* to the domestic market
$EX_{j,x}$:	Quantity of product *x* exported by sector *j*
EXD_x :	World demand for exports of product *x*
IM_m :	Quantity of product *m* imported
INV_i :	Final demand of commodity *i* for investment purposes
$KD_{k,j}$:	Demand for type *k* capital by industry *j*

[5] Source: Decaluwé et al. (2009).

KDC_j :	Industry *j* demand for composite capital
KS_k :	Supply of type *k* capital
$LD_{l,j}$:	Demand for type *l* labor by industry *j*
LDC_j:	Industry *j* demand for composite labor
LS_l :	Supply of type *l* labor
$MRGN_i$:	Demand for commodity *i* as a trade or transport margin
Q_i :	Quantity demanded of composite commodity *i*
VA_j :	Value added of industry *j*
$VSTK_i$:	Inventory change of commodity *i*
$XS_{j,i}$:	Industry *j* production of commodity *i*
XST_j:	Total aggregate output of industry *j*

Price variables

e :	Exchange rate[11]; price of foreign currency in terms of local currency
$P_{j,i}$:	Basic price of industry *j*'s production of commodity *i*
PC_i :	Purchaser price of composite comodity *i* (including all taxes and margins)
PCI_j:	Intermediate consumption price index of industry *j*
PD_i :	Price of local product *i* sold on the domestic market (including all taxes and margins)
PE_x :	Price received for exported commodity *x* (excluding export taxes)
PE_x^{FOB}:	Price of exported commodity *x* (in local currency)
$PIXCON$:	Consumer price index
$PIXGDP$:	GDP deflator
$PIXGVT$:	Public expenditures price index
$PIXINV$:	Investment price index
PL_i :	Price of local product *i* (excluding all taxes on products)
PM_m :	Price of imported product *m* (including all taxes and tariffs)
PP_j :	Industry *j* unit cost, including taxes directly related to the use of capital and labor, but excluding other taxes on production
PT_j :	Basic price of industry *j*'s output
PVA_j :	Price of industry *j* value added (including taxes on production directly related to the use of capital and labour)
PWM_m :	World price of imported product *m* (expressed in foreign currency)
PWX_x :	World price of exported product *x* (expressed in foreign currency)
$R_{k,j}$:	Rental rate of type *k* capital in industry *j*
RC_j :	Rental rate of industry *j* composite capital
RK_k :	Rental rate of type *k* capital (if capital is mobile)
$RTI_{k,j}$:	Rental rate paid by industry *j* for type *k* capital, including capital taxes
W_l :	Wage rate of type *l* labor
WC_j :	Wage rate of industry *j* composite labor
$WTI_{l,j}$:	Wage rate paid by industry *j* for type *l* labor, including payroll taxes

Nominal (value) variables

CAB :	Current account balance
CTH_h :	Consumption budget of type *h* households
G :	Current government expenditures on goods and services
GDP^{BP} :	GDP at basic prices
GDP^{FD} :	GDP at purchasers' prices from the perspective of final demand
GDP^{IB} :	GDP at market prices (income-based)
GDP^{MP} :	GDP at market prices
$GFCF$:	Gross fixed capital formation

IT : Total investment expenditures
SF_f : Savings of type *f* businesses
SG : Government savings
SH_h : Savings of type *h* households
$SROW$: Rest-of-the-world savings
TDF_f : Income taxes of type *f* businesses
$TDFT$: Total government revenue from business income taxes
TDH_h : Income taxes of type *h* households
$TDHT$: Total government revenue from household income taxes
TIC_i : Government revenue from indirect taxes on product *i*
$TICT$: Total government receipts of indirect taxes on commodities
$TIK_{k,j}$: Government revenue from taxes on type *k* capital used by industry *j*
$TIKT$: Total government revenue from from taxes on capital
TIM_m : Government revenue from import duties on product *m*
$TIMT$: Total government revenue from import duties
TIP_j : Government revenue from taxes on industry *j* production (excluding taxes directly related to the use of capital and labor)
$TIPT$: Total government revenue from production taxes (excluding taxes directly related to the use of capital and labor)
$TIW_{l,j}$: Government revenue from payroll taxes on type *l* labor in industry *j*
$TIWT$: Total government revenue from payroll taxes
TIX_x : Government revenue from export taxes on product *x*
$TIXT$: Total government revenue from export taxes
$TPRCTS$: Total government revenue from taxes on products and imports
$TPRODN$: Total government revenue from other taxes on production12
$TR_{ag,agj}$: Transfers from agent *agj* to agent *ag*
YDF_f : Disposable income of type *f* businesses
YDH_h : Disposable income of type *h* households
YF_f : Total income of type *f* businesses
YFK_f : Capital income of type *f* businesses
$YFTR_f$: Transfer income of type *f* businesses
YG : Total government income
YGK : Government capital income
$YGTR$: Government transfer income
YH_h : Total income of type *h* households
YHK_h : Capital income of type *h* households
YHL_h : Labor income of type *h* households
$YHTR_h$: Transfer income of type *h* households
$YROW$: Rest-of-the-world income

Parameters

$aij_{i,j}$: Input-output coefficient
B_j^{KD} : Scale parameter (CES – composite capital)
B_j^{LD} : Scale parameter (CES – composite labor)
B_m^M : Scale parameter (CES – composite commodity)
B_j^{VA} : Scale parameter (CES – value added)
$B_{j,x}^X$: Scale parameter (CET – exports and local sales)
B_j^{XT} : Scale parameter (CET – total output)
$\beta_{k,j}^{KD}$: Share parameter (CES – composite capital)
$\beta_{l,j}^{LD}$: Share parameter (CES – composite labor)
β_m^M : Share parameter (CES – composite commodity)
β_j^{VA} : Share parameter (CES – value added)
$\beta_{j,x}^X$: Share parameter (CET – exports and local sales)

$\beta_{j,i}^{XT}$: Share parameter (CET – total output)
η : Price elasticity of indexed transfers and parameters
γ_i^{GVT} : Share of commodity *i* in total current public expenditures on goods and services
γ_i^{INV} : Share of commodity *i* in total investment expenditures
$\gamma_{i,h}^{LES}$: Marginal share of commodity *i* in type *h* household consumption budget
io_j : Coefficient (Leontief – intermediate consumption)
$\lambda_{ag,k}^{RK}$: Share of type *k* capital income received by agent *ag*
$\lambda_{ag,agj}^{TR}$: Share parameter (transfer functions)
$\lambda_{h,l}^{WL}$: Share of type *l* labor income received by type *h* households
ρ_j^{KD} : Elasticity parameter (CES – composite capital); $-1 < \rho_j^{KD} < \infty$
ρ_j^{LD} : Elasticity parameter (CES – composite labor); $-1 < \rho_j^{LD} < \infty$
ρ_m^M: Elasticity parameter (CES – composite commodity); $-1 < \rho_m^M < \infty$
ρ_j^{VA} : Elasticity parameter (CES – value added) ; $-1 < \rho_j^{VA} < \infty$
$\rho_{j,x}^X$: Elasticity parameter (CET – exports and local sales) ; $1 < \rho_{j,x}^X < \infty$
ρ_j^{XT}: Elasticity parameter (CET – total output) ; $1 < \rho_j^{XT} < \infty$
σ_j^{KD} : Elasticity of substitution (CES – composite capital); $0 < \sigma_j^{KD} < \infty$
σ_j^{LD}: Elasticity of substitution (CES – composite labor); $0 < \sigma_j^{LD} < \infty$
σ_m^M : Elasticity of substitution (CES – composite commodity); $0 < \sigma_m^M < \infty$
σ_j^{VA} : Elasticity of transformation (CES – value added) ; $0 < \sigma_j^{VA} < \infty$
$\sigma_{j,x}^X$: Elasticity of transformation (CET – exports and local sales) ; $0 < \sigma_{j,x}^X < \infty$
σ_x^{XD} : Price elasticity of the world demand for exports of product *x*
σ_j^{XT} : Elasticity of transformation (CET – total output) ; $0 < \sigma_j^{XT} < \infty$
$sh0_h$: Intercept (type *h* household savings)
$sh1_h$: Slope (type *h* household savings)
$tmrg_{i,ij}$: Rate of margin *i* applied to commodity *ij*
$tmrg_{i,x}^X$: Rate of margin *i* applied to exported commodity *x*
$tr0_h$: Intercept (transfers by type *h* households to government)
$tr1_h$: Marginal rate of transfers by type *h* households to government
$ttdf0_h$: Intercept (income taxes of type *f* businesses)
$ttdf1_h$: Marginal income tax rate of type *f* businesses
$ttdh0_h$: Intercept (income taxes of type *h* households)
$ttdh1_h$: Marginal income tax rate of type *h* households
$ttic_i$: Tax rate on commodity *i*
$ttik_{k,j}$: Tax rate on type *k* capital used in industry *j*
$ttim_m$: Rate of taxes and duties on imports of commodity *m*
$ttip_j$: Tax rate on the production of industry *j*
$ttiw_{l,j}$: Tax rate on type *l* worker compensation in industry *j*
$ttix_x$: Export tax rate on exported commodity *x*
v_j : Coefficient (Leontief – value added)

EQUATIONS

Production

1. $VA_j = v_j XST_j$
2. $CI_j = io_j XST_j$
3. $VA_j = B_j^{VA}\left[\beta_j^{VA} LDC_j^{-\rho_j^{VA}} + \left(1-\beta_j^{VA}\right) KDC_j^{-\rho_j^{VA}}\right]^{-\frac{1}{\rho_j^{VA}}}$

4. $LDC_j = \left[\frac{\beta_j^{VA}}{1-\beta_j^{VA}}\frac{RC_j}{WC_j}\right]^{\sigma_j^{VA}} KDC_j$
5. $LDC_j = B_j^{LD}\left[\sum_l \beta_{l,j}^{LD} LD_{l,j}^{-\rho_j^{LD}}\right]^{-\frac{1}{\rho_j^{LD}}}$
6. $LD_{l,j} = \left[\frac{\beta_{l,j}^{LD} WC_j}{WTI_{l,j}}\right]^{\sigma_j^{LD}} \left(B_j^{LD}\right)^{\sigma_j^{LD}-1} LDC_j$
7. $KDC_j = B_j^{KD}\left[\sum_k \beta_{k,j}^{KD} KD_{k,j}^{-\rho_j^{KD}}\right]^{-\frac{1}{\rho_j^{KD}}}$
8. $KD_{k,j} = \left[\frac{\beta_{k,j}^{KD} WC_j}{RTI_{k,j}}\right]^{\sigma_j^{LD}} \left(B_j^{KD}\right)^{\sigma_j^{KD}-1} KDC_j$
9. $DI_{i,j} = aij_{i,j} CI_j$

Income and savings

Households

10. $YH_h = YHL_h + YHK_h + YHTR_h$
11. $YHL_h = \sum_l \lambda_{h,l}^{WL}\left(W_l \sum_j LD_{l,j}\right)$
12. $YHK_h = \sum_k \lambda_{h,k}^{RK}\left(\sum_j R_{k,j} KD_{k,j}\right)$
13. $YHTR_h = \sum_{ag} TR_{h,ag}$
14. $YDH_h = YH_h - TDH_h - TR_{gvt,h}$
15. $CTH_h = YDH_h - SH_h - \sum_{agng} TR_{agng,h}$
16. $SH_h = PIXCON^{\eta} sh0_h + sh1_h YDH_h$

Firms

17. $YF_f = YFK_f + YFTR_f$
18. $YFK_f = \sum_k \lambda_{f,k}^{RK}\left(\sum_j R_{k,j} KD_{k,j}\right)$
19. $YHTR_f = \sum_{ag} TR_{f,ag}$
20. $YDF_f = YF_f - TDF_f$
21. $SF_f = YDF_f - \sum_{ag} TR_{ag,f}$

Government

22. $YG = YGK + TDHT + TDFT + TPRODN + TPRCTS + YGTR$
23. $YGK = \sum_k \lambda_{gvt,k}^{RK}\left(\sum_j R_{k,j} KD_{k,j}\right)$
24. $TDHT = \sum_h TDH_h$
25. $TDFT = \sum_f TDF_f$
26. $TPRODN = TIWT + TIKT + TIPT$
27. $TIWT = \sum_{l,j} TIW_{l,j}$
28. $TIKT = \sum_{k,j} TIK_{k,j}$
29. $TIPT = \sum_j TIP_j$
30. $TPRCTS = TICT + TIMT + TIXT$
31. $TICT = \sum_i TIC_i$

32. $TIMT = \sum_m TIM_m$

33. $TIXT = \sum_x TIX_x$

34. $YGTR = \sum_{agng} TR_{gvt,agng}$

35. $TDH_h = PIXCON^{\eta} ttd0_h + ttd1_h YH_h$

36. $TDF_f = PIXCON^{\eta} ttd0_f + ttd1_f YFK_f$

37. $TIW_{l,j} = ttiw_{l,j} W_l LD_{l,j}$

38. $TIK_{k,j} = ttik_{k,j} R_{k,j} KD_{k,j}$

39. $TIP_j = ttip_j PP_j XST_j$

40. $TIC_{nm} = ttic_{nm}\left(PL_{nm} + \sum_i PC_i tmrg_{i,nm}\right) DD_{nm}$

41. $TIC_m = ttic_m \left[\left(PL_m + \sum_i PC_i tmrg_{i,m}\right) DD_m + \left((1 + ttim_m) PWM_m e + \sum_i PC_i tmrg_{i,m}\right) IM_m\right]$

42. $TIM_m = ttim_m PWM_m \; e \; IM_m$

43. $TIX_x = ttix_x\left(PE_x + \sum_i PC_i tmrg_{i,x}^X\right) EXD_x$

44. $SG = YG - \sum_{agng} TR_{agng,gvt} - G$

Rest of the world

45. $YROW = e \sum_m PWM_m \, IM_m + \sum_k \lambda_{row,k}^{RK}\left(\sum_j R_{k,j} KD_{k,j}\right) + \sum_{agd} TR_{row,agd}$

46. $SROW = YROW - \sum_x PE_x^{FOB} EXD_x - \sum_{agd} TR_{AGD,row}$

47. $SROW = -CAB$

Transfers

48. $TR_{agng,h} = \lambda_{agng,h}^{TR} YDH_h$

49. $TR_{gvt,h} = PIXCON^{\eta} tr0_h + tr1_h YH_h$

50. $TR_{ag,f} = \lambda_{ag,f}^{TR} YDF_f$

51. $TR_{agng,gvt} = PIXCON^{\eta} TR_{agng,gvt}^0$

52. $TR_{agd,row} = PIXCON^{\eta} TR_{agd,row}^0$

TR

Demand

53. $C_{i,h} PC_i = C_{i,h}^{MIN} PC_i + \gamma_{i,h}^{LES}\left(CTH_h - \sum_{ij} C_{ij,h}^{MIN} PC_{ij}\right)$

54. $GFCF = IT - \sum_i PC_i VSTK_i$

55. $PC_i INV_i = \gamma_i^{INV} GFCF$

56. $PC_i CG_i = \gamma_i^{GVT} G$

57. $DIT_i = \sum_j DI_{i,j}$

58. $MRGN_i = \sum_{ij} tmrg_{i,ij} DD_{ij} + \sum_m tmrg_{i,m} IM_m + \sum_x tmrg_{i,x}^X EXD_x$

Producer supplies of products and international trade

59. $XST_j = B_j^{XT}\left[\sum_i \beta_{j,i}^{XT} XS_{j,i}^{\rho_j^{XT}}\right]^{\frac{1}{\rho_j^{XT}}}$

60. $XS_{j,i} = \frac{XST_j}{\left(B_j^{XT}\right)^{1+\sigma_j^{XT}}}\left[\frac{P_{j,i}}{B_{j,i}^{XT} PT_j}\right]^{\sigma_j^{XT}}$

61. $XS_{j,x} = B_{j,x}^X\left[\beta_{j,x}^X EX_{j,x}^{\rho_{j,x}^X} + \left(1-\beta_{j,x}^X\right)DS_{j,x}^{\rho_{j,x}^X}\right]^{\frac{1}{\rho_{j,x}^X}}$

62. $XS_{j,nx} = DS_{j,nx}$

63. $EX_{j,x} = \left[\frac{1-\beta_{j,x}^X}{\beta_{j,x}^X}\frac{PE_x}{PL_x}\right]^{\sigma_{j,x}^X} DS_{j,x}$

64. $EXD_x = EXD_x^0\left[\frac{ePWX_x}{PE_x^{FOB}}\right]^{\sigma_x^{XD}}$

65. $QM_m = B_m^M\left[\beta_m^M IM_m^{-\rho_m^M} + (1-\beta_m^M)DD_m^{-\rho_m^M}\right]^{-\frac{1}{\rho_m^M}}$

66. $Q_{nm} = DD_{nm}$

67. $IM_m = \left[\frac{\beta_m^M}{1-\beta_m^M}\frac{PD_m}{PM_m}\right]^{\sigma_m^M} DD_m$

Prices

Production

68. $PP_j = \frac{PVA_j VA_j + PCI_j CI_j}{XST_j}$

69. $PT_j = \left(1+ttip_j\right)PP_j$

70. $PCI_j = \frac{\sum_i PCI_i DI_{i,j}}{CI_j}$

71. $PVA_j = \frac{WC_j LDC_j + RC_j KDC_j}{VA_j}$

72. $WC_j = \frac{\sum_l WTI_{l,j} LD_{l,j}}{LDC_j}$

73. $WTI_{l,j} = W_l\left(1+ttiw_{l,j}\right)$

74. $RC_j = \frac{\sum_k RTI_{k,j} KD_{k,j}}{KDC_j}$

75. $RTI_{k,j} = R_{k,j}\left(1+ttik_{k,j}\right)$

76. $R_{k,j} = RK_k$ if capital is mobile

International trade

77. $PT_j = \frac{\sum_i P_{j,i} XS_{j,i}}{XST_j}$

78. $P_{j,x} = \frac{PE_x EX_{j,x} + PL_x DS_{j,x}}{XS_{j,x}}$

79. $P_{j,nx} = PL_{nx}$

80. $PE_x^{FOB} = \left(PE_x + \sum_i PC_i tmrg_{i,x}^X\right)(1+ttix_x)$

81. $PD_i = (1+ttic_i)\left(PL_i + \sum_{ij} PC_i tmrg_{ij,i}\right)$

82. $PM_m = (1+ttic_m)\left((1+ttim_m)ePWM_m + \sum_i PC_i tmrg_{i,m}\right)$

83. $PC_m = \frac{PM_m IM_m + PD_m DD_m}{QM_m}$

84. $PC_{nm} = PD_{nm}$

Price indexes

85. $PIXGDP = \sqrt{\frac{\sum_j PVA_j VA0_j}{\sum_j PVA0_j VA0_j} \frac{\sum_j PVA_j VA_j}{\sum_j PVA0_j VA_j}}$

86. $PIXCON = \frac{\sum_i PC_i \sum_h C^0_{i,h}}{\sum_{ij} PC^0_{ij} \sum_h C^0_{ij,h}}$

87. $PIXINV = \prod_i \left(\frac{PC_i}{PC^0_i}\right)^{\gamma^{INV}_i}$

88. $PIXGVT = \prod_i \left(\frac{PC_i}{PC^0_i}\right)^{\gamma^{GVT}_i}$

Equilibrium

89. $Q_i = \sum_h C_{i,h} + CG_i + INV_i + VSTK_i + DIT_i + MRGN_i$

90. $\sum_j LD_{l,j} = LS_l$

91. $\sum_j KD_{k,j} = KS_k$

92. $IT = \sum_h SH_h + \sum_f SF_f + SG + SROW$

93. $\sum_j DS_{j,i} = DD_i$

94. $\sum_j EX_{j,x} = EXD_x$

Gross domestic product

95. $GDP^{BP} = \sum_j PVA_j VA_j + TIPT$

96. $GDP^{MP} = GDP^{BP} + TPRCTS$

97. $GDP^{IB} = \sum_{l,j} W_l LD_{l,j} + \sum_{k,j} R_{k,j} KD_{k,j} + TPRODN + TPRCTS$

98. $GDP^{FD} = \sum_i PC_i \left[\sum_h C_{i,h} + CG_i + INV_i + VSTK_i\right] + \sum_x PE^{FOB}_x EXD_x - ePWM_m IM_m$

Appendix C: The IFPRI standard model[6]

SETS

$\alpha \in A$	activities
$\alpha \in ACES (\subset A)$	activities with a CES function at the top of the technology nest
$\alpha \in ALEO (\subset A)$	activities with a Leontief function at the top of the technology nest
$c \in C$	commodities
$c \in CD (\subset C)$	commodities with domestic sales of domestic output
$c \in CDN (\subset C)$	commodities not in CD
$c \in CE (\subset C)$	exported commodities
$c \in CEN (\subset C)$	commodities not in CE
$c \in CM (\subset C)$	imported commodities
$c \in CMN (\subset C)$	commodities not in CM
$c \in CT (\subset C)$	transactions service commodities
$c \in CX (\subset C)$	commodities with domestic production
$f \in F$	factors
$i \in INS$	institutions (domestic and rest of the world)
$i \in INSD (\subset INS)$	domestic institutions
$I \in INSDNG (\subset INSD)$	domestic nongovernment institutions
$h \in H (\subset INSDNG)$	households

PARAMETERS

$cwts_c$	weight of commodity c in the CPI
$dwts_c$	weight of commodity c in the producer price index
$ica_{c\,a}$	quantity of c as intermediate input per unit of activity a
$icd_{c\,c'}$	quantity of commodity c as trade input per unit of c' produced and sold domestically
$ice_{c\,c'}$	quantity of commodity c as trade input per exported unit of c'
$icm_{c\,c'}$	quantity of commodity c as trade input per imported unit of c'
$inta_a$	quantity of aggregate intermediate input per activity unit
iva_a	quantity of value-added per activity unit
$\overline{mps}$	base savings rate for domestic institution i
$mps01_c$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
pwe_c	export price (foreign currency)
pwm_c	import price (foreign currency)
$qdst_c$	quantity of stock change
$\overline{qg}_c$	base-year quantity of government demand
$\overline{qinv}_c$	base-year quantity of private investment demand
$shif_{if}$	share for domestic institution i in income of factor
$shii_{i\,i'}$	share of net income of i' to i ($i' \in INSDNG'$; $i \in INSDNG$)
ta_a	tax rate for activity a
te_c	export tax rate
tff	direct tax rate for factor f
$\overline{tins}$	exogenous direct tax rate for domestic institution i
$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates

[6] Source: Lofgren et al. (2002).

tm_C	import tariff rate
tq_C	rate of sales tax
$trnsfr_{i\,f}$	transfer from factor *f* to institution *i*
tva_a	rate of value-added tax for activity
α_a^a	efficiency parameter in the CES activity function
α_a^{va}	efficiency parameter in the CES value-added function
α_a^{ac}	shift parameter for domestic commodity aggregation function
α_c^q	Armington function shift parameter
α_c^t	CET function shift parameter
β_{ach}^h	marginal share of consumption spending on home commodity c from activity a for household h
β_{ch}^m	marginal share of consumption spending on marketed commodity c for household h
δ_a^a	CES activity function share parameter
$\delta_{a\,c}^{ac}$	share parameter for domestic commodity aggregation function
δ_c^q	Armington function share parameter
δ_c^t	CET function share parameter
δ_{fa}^{va}	CES value-added function share parameter for factor f in activity a
γ_{ch}^m	subsistence consumption of marketed commodity c for household h
γ_{ach}^h	subsistence consumption of home commodity c from activity a for household h
$\theta_{a\,c}$	yield of output c per unit of activity a
ρ_a^a	CES production function exponent
ρ_a^{va}	CES value-added function exponent
ρ_c^{ac}	Domestic commodity aggregation function exponent
ρ_c^q	Armington function exponent
ρ_c^t	CET function exponent

EXOGENOUS VARIABLES

$\overline{CPI}$	consumer price index
$\overline{DTINS}$	change in domestic institution tax share (= 0 for base; exogenous variable)
$\overline{FSAV}$	foreign savings (FCU)
$\overline{GADJ}$	government consumption adjustment factor
$\overline{IADJ}$	investment adjustment factor
$\overline{MPSADJ}$	savings rate scaling factor (= 0 for base)
$\overline{QFS}_f$	quantity supplied of factor
$\overline{TINSADJ}$	direct tax scaling factor (= 0 for base; exogenous variable)
$\overline{WFDIST}_{fa}$	wage distortion factor for factor f in activity a

ENDOGENOUS VARIABLES

DMPS	change in domestic institution savings rates (= 0 for base; exogenous variable)
DPI	producer price index for domestically marketed output
EG	government expenditures
EH_h	consumption spending for household
EXR	exchange rate (*LCU* per unit of *FCU*)
GOVSHR	government consumption share in nominal absorption
GSAV	government savings
INVSHR	investment share in nominal absorption

MPS_i	marginal propensity to save for domestic non- government institution (exogenous variable)
PA_a	activity price (unit gross revenue)
PDD_c	demand price for commodity produced and sold domestically
PDS_c	supply price for commodity produced and sold domestically
PE_c	export price (domestic currency)
$PINTA_a$	aggregate intermediate input price for activity a
PM_c	import price (domestic currency)
PQ_c	composite commodity price
PVA_a	value-added price (factor income per unit of activity)
PX_c	aggregate producer price for commodity
$PXAC_{a\ c}$	producer price of commodity c for activity a
QA_a	quantity (level) of activity
QD_c	quantity sold domestically of domestic output
QE_c	quantity of exports
$QF_{f\ a}$	quantity demanded of factor f from activity a
QG_c	government consumption demand for commodity
$QH_{c\ h}$	quantity consumed of commodity c by household h
$QHA_{a\ c\ h}$	quantity of household home consumption of commodity c from activity a for household h
$QINTA_a$	quantity of aggregate intermediate input
$QINT_{c\ a}$	quantity of commodity c as intermediate input to activity a
$QINV_c$	quantity of investment demand for commodity
QM_c	quantity of imports of commodity
QQ_c	quantity of goods supplied to domestic market (composite supply)
QT_c	quantity of commodity demanded as trade input
QVA_a	quantity of (aggregate) value-added
QX_c	aggregated marketed quantity of domestic output of commodity
$QXAC_{a\ c}$	quantity of marketed output of commodity c from activity a
$TABS$	total nominal absorption
$TINS_i$	direct tax rate for institution i ($i \in INSDNG$)
$TRII_{i\ i'}$	transfers from institution i' to i (both in the set INSDNG)
WF_f	average price of factor f
YF_f	income of factor f
YG	government revenue
YI_i	income of domestic nongovernment institution
$YIF_{i\ f}$	income to domestic institution i from factor f

EQUATIONS

Price Block

$$PM_c = pwm_c \cdot (1+tm_c) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c'\ c} \qquad c \in CM \qquad (1)$$

$$PE_c = pwe_c \cdot (1-te_c) \cdot EXR - \sum_{c' \in CT} PQ_{c'} \cdot ice_{c'\ c} \qquad c \in CE \qquad (2)$$

$$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c'\ c} \qquad c \in CD \qquad (3)$$

$$PQ_c \cdot (1-tq_c) \cdot QQ_c = PDD_c \cdot QD_c + PM_c \cdot QM_c \qquad c \in (CD \cup CM) \qquad (4)$$

$$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c \qquad c \in CX \qquad (5)$$

$$PA_a = \sum_{c \in C} PXAC_{a\,c} \cdot \theta_{a\,c} \qquad a \in A \qquad (6)$$

$$PINTA_a = \sum_{c \in C} PQ_c \cdot ica_{c\,a} \qquad a \in A \qquad (7)$$

$$PA_a \cdot (1 - ta_a) \cdot QA_a = PVA_a \cdot QVA_a + PINTA_a \cdot QINTA_a \qquad a \in A \qquad (8)$$

$$\overline{CPI} = \sum_{c \in C} PQ_c \cdot cwts_c \qquad (9)$$

$$DPI = \sum_{c \in C} PDS_c \cdot dwts_c \qquad (10)$$

Production and Trade Block

$$QA_a = \alpha_a^a \cdot \left(\delta_a^a \cdot QVA_a^{-\rho_a^a} + (1 - \delta_a^a) \cdot QINTA_a^{-\rho_a^a}\right)^{-\frac{1}{\rho_a^a}} \qquad a \in ACES \qquad (11)$$

$$\frac{QVA_a}{QINTA_a} = \left(\frac{PINTA_a}{PVA_a} \cdot \frac{\delta_a^a}{1 - \delta_a^a}\right)^{\frac{1}{1+\rho_a^a}} \qquad a \in ACES \qquad (12)$$

$$QVA_a = iva_a \cdot QA_a \qquad a \in ALEO \qquad (13)$$

$$QINTA_a = inta_a \cdot QA_a \qquad a \in ALEO \qquad (14)$$

$$QINT_{c\,a} = ica_{c\,a} \cdot QINTA_a \qquad a \in A \quad c \in C \qquad (17)$$

$$QXAC_{a\,c} + \sum_{h \in H} QHA_{a\,c\,h} = \theta_{a\,c} \cdot QA_a \qquad a \in A \quad a \in CX \qquad (18)$$

$$QX_c = \alpha_a^{ac} \cdot \left(\sum_{a \in A} \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_a^{ac}}\right)^{-\frac{1}{\rho_a^{ac}-1}} \qquad c \in CX \qquad (19)$$

$$PXAC_{a\,c} = PX_c \cdot QX_c \left(\sum_{a \in A'} \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_a^{ac}}\right)^{-1} \cdot \delta_{a\,c}^{ac} \cdot QXAC_{a\,c}^{-\rho_a^{ac}-1} \qquad a \in A \quad c \in CX \qquad (20)$$

$$QX_c = \alpha_c^t \cdot \left(\delta_c^t \cdot QE_c^{\rho_c^t} + (1 - \delta_c^t) \cdot QD_c^{\rho_c^t}\right)^{\frac{1}{\rho_c^t}} \qquad c \in (CE \cap CD) \qquad (21)$$

$$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \cdot \frac{1 - \delta_c^t}{\delta_c^t}\right)^{\frac{1}{\rho_c^t}-1} \qquad c \in (CE \cap CD) \qquad (22)$$

$$QX_c = QD_c + QE_c \qquad c \in (CD \cap CEN) \cup (CE \cup CDN) \qquad (23)$$

$$QQ_c = \alpha_c^q \cdot \left(\delta_c^q \cdot QM_c^{-\rho_c^q} + \left(1 - \delta_c^q\right) \cdot QD_c^{-\rho_c^q}\right)^{-\frac{1}{\rho_c^q}} \qquad c \in (CM \cap CD) \qquad (24)$$

$$\frac{QM_c}{QD_c} = \left(\frac{PDD_c}{PM_c} \cdot \frac{\delta_c^q}{1 - \delta_c^q}\right)^{\frac{1}{1+\rho_c^q}} \qquad c \in (CM \cap CD) \qquad (25)$$

$$QQ_c = QD_c + QM_c \qquad c \in (CD \cap CMN) \cup (CM \cup CDN) \qquad (26)$$

$$QT_c = \sum_{c' \in C'} \left(icm_{c\,c'} \cdot QM_{c'} + ice_{c\,c'} \cdot QE_{c'} + icd_{c\,c'} \cdot QD_{c'}\right) \qquad c \in CT \qquad (27)$$

Institution Block

$$YF_f = \sum_{a \in A} WF_f \cdot \overline{WFDIST}_{f\,a} \cdot QF_{f\,a} \qquad f \in F \qquad (28)$$

$$YIF_{i\,f} = shif_{i\,f} \cdot \left[\left(1 - tf_f\right) \cdot YF_f - trnsfr_{row\,f} \cdot EXR\right] \qquad i \in INSD \quad f \in F \qquad (29)$$

$$YI_i = \sum_{f \in F} YIF_{i\,f} + \sum_{i' \in INSDNG} TRII_{i\,i'} + trnsfr_{i\,gov} \cdot \overline{CPI} + trnsfr_{i\,row} \cdot EXR \qquad i \in INSDNG \qquad (30)$$

$$TRII_{i\,i'} = shii_{i\,i'} \cdot (1 - MPS_{i'}) \cdot (1 - TINS_{i'}) \cdot YI_{i'} \qquad i \in INSDNG \quad i' \in INSDNG' \qquad (31)$$

$$EH_h = \left(1 - \sum_{i \in INSDNG} shii_{i\,h}\right) \cdot (1 - MPS_h) \cdot (1 - TINS_h) \cdot YI_h \qquad h \in H \qquad (32)$$

$$PQ_c \cdot QH_{c\,h} = PQ_c \cdot \gamma^m_{ch} + \beta^m_{ch} \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot \gamma^m_{c'h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a\,c'} \cdot \gamma^h_{ac'h}\right) \qquad c \in C \;\; h \in H \qquad (33)$$

$$PXAC_{a\,c} \cdot QHA_{ach} = PXAC_{ac} \cdot \gamma^h_{ach} + \beta^h_{ach} \cdot (EH_h - \sum PQ_{c'} \cdot \gamma^m_{c'h} - \sum_{a \in A} \sum_{c' \in C} PXAC_{ac'} \cdot \gamma^h_{ac'h}) \quad a \in A \;\; c \in C \;\; h \in H \qquad (34)$$

$$QINV_c = \overline{IADJ} \cdot \overline{qinv_c} \qquad c \in CINV \qquad (35)$$

$$QG_c = GADJ \cdot \overline{qg_c} \qquad c \in C \qquad (36)$$

$$\begin{aligned} YG = {} & \sum_{i \in INSDNG} TINS_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{a \in A} tva_a \cdot PVA_a \cdot QVA_a + \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a \\ & + \sum_{c \in CM} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{c \in CE} te_c \cdot pwe_c \cdot QE_c \cdot EXR \\ & + \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YIF_{gov\,f} + trnsfr_{gov\,row} \cdot EXR \end{aligned} \qquad (37)$$

$$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in INSDNG} trnsfr_{i\,gov} \cdot \overline{CPI} \qquad (38)$$

System Constraint Block

$$\sum_{a \in A} QF_{f\,a} = QFS_f \qquad f \in F \qquad (39)$$

$$QQ_c = \sum_{a \in A} QINT_{c\,a} + \sum_{h \in H} QH_{c\,h} + QG_c + QINV_c + qdst_c + QT_c \qquad c \in C \qquad (40)$$

$$\sum_{c \in CM} pwm_c \cdot QM_c + \sum_{f \in F} trnsfr_{row\,f} = \sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in INSD} trnsfr_{i\,row} + \overline{FSAV} \qquad (41)$$

$$YG = EG + GSAV \qquad (42)$$

$$TINS_i = tins_i \cdot \left(1 + TINSADJ \cdot tins01_i\right) + DTINS \cdot tins01_i \qquad i \in INSDNG \qquad (43)$$

$$MPS_i = \overline{mps_i} \cdot \left(1 + \overline{MPSADJ} \cdot mps01_i\right) + DMPS \cdot mps01_i \qquad i \in INSDNG \qquad (44)$$

$$\sum_{i \in INSDNG} MPS_i \cdot (1 - TINS_i) \cdot YI_i + GSAV + EXR \cdot \overline{FSAV} = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \qquad (45)$$

$$TABS = \sum_{h \in H} \sum_{c \in C} PQ_c \cdot QH_{c\,h} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{a\,c} \cdot QHA_{a\,c\,h} + \sum_{c \in C} PQ_c \cdot QG_c + \sum_{c \in C} PQ_c \cdot QINV_c + \sum PQ_c \cdot qdst_c \qquad (46)$$

$$INVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot INV_c + \sum_{c \in C} PQ_c \cdot qdst_c \qquad (47)$$

$$GOVSHR \cdot TABS = \sum_{c \in C} Q_c \cdot QG_c \qquad (48)$$